Substituting this value of x in the last two fractions,

$$\frac{dz}{a}=\frac{dy}{1+cy},\ \text{or}\ z=\frac{a}{c}\log(1+cy)+c'.$$

Replacing c by $\frac{1}{x}-\frac{1}{y}$, $z=\frac{axy}{y-x}\log\frac{y}{x}+\phi\left(\frac{1}{x}-\frac{1}{y}\right)$.

Also solved similarly by J. Scheffer, and G. B. M. Zerr.

293. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find the length of the integral curve of the differential equation $(y^2x^{\frac{4}{3}}+2)dx-x^{\frac{4}{3}}dy=0$ between $x_1=1$ and $x_2=8$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $y=-1/z$, $x^{-\frac{1}{3}}=v$, then the equation becomes $dz+6z^2dv+3v^{-4}dv=0$.

Now, let $z=\frac{1}{6v}+\frac{u}{v^2}$.

Then $v^2du+6u^2dv+3dv=0$, or $\frac{du}{3+6u^2}=-\frac{dv}{v^2}$.

$$\therefore \frac{1}{3\sqrt{2}}\tan^{-1}(u\sqrt{2})=\frac{1}{v}+a=a+x^{\frac{1}{3}} \ldots (1).$$

$$\therefore u=\frac{1}{\sqrt{2}}\tan[3\sqrt{2}(a+x^{\frac{1}{3}})] \ldots (2).$$

$$\therefore y=-\frac{6\sqrt{2}}{6x^{\frac{2}{3}}\tan[3\sqrt{2}(a+x^{\frac{1}{3}})]+x^{\frac{1}{3}}\sqrt{2}}\ \text{is the equation.}$$

From (1), $u=\frac{1}{\sqrt{2}}\tan\left[\frac{3\sqrt{2}(av+1)}{v}\right]$.

$$S=\int\sqrt{[1+(du/dv)^2]}\,dv=\int_{\frac{1}{2}}^{1}\frac{1}{v^2}\sqrt{v^4+9\sec^4\left[\frac{3\sqrt{2}(av+1)}{v}\right]}\,dv.$$

294. Proposed by C. N. SCHMALL, New York City.

Examine the function, $f(x)=\frac{(x-1)(x-2)}{(x-3)}$ and determine why its *minimum* value is *greater* than its maximum.